On certain Geological Effects of the Cooling of the Earth.

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1. *The Thermal History of the Earth.*

The former molten state of the earth has been widely accepted as an important and fruitful hypothesis in physical geology ever since the enunciation of Laplace's theory of the origin of the solar system. Elie de Beaumont, followed by other investigators, suggested that the cooling and consequent contraction of the crust was a potent cause, and probably the chief cause, of the horizontal compression that led to mountain building. The theory was put into quantitative form by C. Davison* and Sir G. H. Darwin,† their numerical data being taken from Lord Kelvin's theory of the cooling of the earth. Osmond Fisher‡ objected to this that the compression they found was inadequate to account for existing mountain chains, but neither he nor those geologists who have followed him appear to have given an estimate of any significance of the amount of compression actually needed. Such an estimate, admittedly rough and provisional, but probably correct in order of magnitude, was made by the writer in a paper§ in the 'Philosophical Magazine' for 1916. At the same time, I used Holmes's data|| about the age and thermal properties of the earth in preference to Kelvin's, and found that with these the available compression appeared to be sufficient.

According to the form of the tidal theory of the origin of the solar system, developed by Chamberlin and Moulton in their Planetesimal Theory, the earth became solidified by adiabatic expansion immediately on its ejection from the sun, and in its subsequent growth by accretion never attained fusion temperature; so that we have to contemplate an earth that has always been solid, and cannot have cooled to anything like the extent that was implied by the older theory. I believe, however, that this supposition is erroneous. It is at least probable that most of the matter that went to form the earth came from the superficial regions of the sun (if the encounter with the passing star was "slow" in Jeans's sense, the whole of it would), and that, when it gathered together, the relative increase in

* 'Phil. Trans.,' A, vol. 178, pp. 231–242 (1887).

† 'Phil. Trans.,' A, vol. 178, pp. 242–249 (1887); or 'Scientific Papers,' vol. 4, p. 354.

‡ 'Physics of the Earth's Crust,' Macmillan, 1889.

§ "The Compression of the Earth's Crust in Cooling," 'Phil. Mag.,' vol. 32, pp. 575–591 (1916).

|| 'Geological Magazine,' March, 1915, pp. 102–112.

the depth was greater than the relative diminution in gravity. If this were so, the average pressure inside this matter would be greater than before instead of less, and the temperature would rise on ejection instead of falling, even if the change was adiabatic. Again, even if it were shown that the mean pressure would fall, adiabatic cooling below the boiling point could be caused only by evaporation, and therefore would not lower the temperature below a point at which the vapour pressure was insignificant. Thus the temperature could never be reduced in this way by more than 200° C. at most below the boiling point. But the difference between the melting and boiling points of the substances concerned is at least several hundred degrees. Hence the primeval earth, if adiabatic cooling took place at all, could at most have cooled to the liquid state and not to the solid state. Solidification must have taken place later and more gradually in consequence of radiation.

As I have shown in a previous paper,* accretion of small particles to such an extent as to be of cosmogonical importance is impossible, since the mutual impacts of the particles would volatilise them before any important growth could take place. Accordingly, we must suppose that at an early stage in the history of the earth it was liquid at the surface, and that subsequent development took place by radiative cooling without much mechanical interference from outside. The theory of a formerly molten and cooling earth is therefore implied by the tidal theory of the origin of the solar system, which, on astronomical grounds, is the most acceptable yet advanced.†

In the present paper I develop certain consequences of the cooling of the earth that have not yet been dealt with. In the first place, however, some discussion of the earth's thermal history is necessary.

The problem of the flow of heat in a solid depends on the differential equation

$$\frac{\partial V}{\partial t} - \frac{k}{\sigma\rho}\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}\right) = \frac{P}{\sigma\rho}, \tag{1}$$

where V is the temperature, t the time, x, y and z the three Cartesian position co-ordinates, k the thermal conductivity, ρ the density, σ the specific heat, and P the number of calories generated in unit time by radio-activity or chemical change in unit volume of the solid. In the present problem the changes in temperature are confined to the superficial layers,

* 'Monthly Notices of Roy. Astron. Soc.,' vol. 77, pp. 84–112 (1916).

† J. H. Jeans, 'Problems of Cosmogony,' Camb. Univ. Press, 1920; H. Jeffreys, "The Early History of the Solar System," 'Monthly Notices of R.A.S.,' vol. 78, pp. 424–441 (1918).

whose thickness is small compared with the radius of the earth. Accordingly, if we take the axis of x to be vertically downwards, the equation reduces to

$$\frac{\partial V}{\partial t} - h^2 \frac{\partial^2 V}{\partial x^2} = \frac{P}{\sigma\rho}, \tag{2}$$

where h^2 has been written for $k/\sigma\rho$. Let some particular integral of this differential equation be V_0. As a rule, P is, with sufficient accuracy, a function of x alone, so that a particular integral is $-\iint P/k\,dx\,dx$. Whether it is or not, however, provided only that a particular integral can be found, we have only to find a solution of the differential equation (with P on the right put equal to zero) which will reduce when t is zero to $f(x)$, where the initial temperature is given to be $f(x)+V_0$, and which will satisfy the appropriate condition at the surface of the earth.

In the present problem the initial temperature is specified from $x = 0$ to $x = \infty$; it is to be derived from the physical conditions at the time of the solidification of the earth. It is easy to show from known data that the amount of heat arriving at the surface of the earth by conduction from the interior is a very small fraction of that obtained by radiation from the sun, and hence that the surface temperature is determined solely by the condition that the radiation into space from the earth must balance that received from the sun. There is no reason to believe that there has been in geological time any wide variation in either the solar radiation or in the radiative constants of the surface of the earth, and consequently we are justified here in assuming that the variations of surface temperature have not been sufficient to demand their recognition. It will be convenient, therefore, to adopt a constant surface temperature for our zero of temperature. Then we have the surface condition that when $x = 0$, the temperature is always zero, except at the initial moment. Now we can, without loss of generality, make the particular integral V_0 one that vanishes with x. Then $V-V_0$ still satisfies the equation (2) after zero has been substituted for P, and is equal to $f(x)$ when t is zero. Also, it vanishes when $x = 0$ for all values of the time. The solution that satisfies all these conditions can be shown to be

$$V = V_0 + \frac{1}{\sqrt{(\pi)}} \int_{-x/2ht^{\frac{1}{2}}}^{\infty} e^{-q^2} f(2qht^{\frac{1}{2}}+x)\,dq - \frac{1}{\sqrt{(\pi)}} \int_{-\infty}^{-x/2ht^{\frac{1}{2}}} e^{-q^2} f(-2qht^{\frac{1}{2}}-x)\,dq. \tag{3}$$

This is readily transformed into

$$V = V_0 + \frac{1}{\sqrt{(\pi)}} \int_0^{\infty} \{e^{-(q-\lambda)^2} - e^{-(q+\lambda)^2}\} f(2qht^{\frac{1}{2}})\,dq, \tag{4}$$

where λ has been written for $x/2ht^{\frac{1}{2}}$. This is the solution of the most general problem of the class considered.

In the problem of the cooling of the earth a good representation of the known data is given* by assuming $P = Ae^{-ax}$, where A and a are constants. Then V_0 is $\frac{A}{a^2k}(1-e^{-ax})$. The initial temperature at any point would be the melting point of the rocks at that depth under the pressure there. This may be taken to be of the form $S+mx$. Then

$$f(x) = S+mx-(A/a^2k)(1-e^{-ax}) \tag{5}$$

and the problem can be solved. We find†

$$V = (A/a^2k)(1-e^{-ax})+mx+(S-A/a^2k)\operatorname{Erf}\lambda$$
$$+(A/2a^2k)\,e^{\gamma^2}[e^{-ax}\{1-\operatorname{Erf}(\gamma-\lambda)\}-e^{ax}\{1-\operatorname{Erf}(\gamma+\lambda)\}] \tag{6}$$

where
$$\gamma = aht^{\frac{1}{2}}, \tag{7}$$

and
$$\lambda = x/2ht^{\frac{1}{2}}. \tag{8}$$

We shall assume that at the present time the second line is small in comparison with the first. Differentiating with regard to x, and then putting x equal to zero, we find that the rate of increase of temperature with depth at the surface is given by

$$\left(\frac{\partial V}{\partial x}\right)_0 = m+\frac{S}{h\sqrt{(\pi t)}}+\frac{A}{ak}\left(1-\frac{1}{ah\sqrt{(\pi t)}}\right). \tag{9}$$

Let us now consider the cooling of a continental region. The average value of $\left(\frac{\partial V}{\partial x}\right)_0$, as determined by borings, is about 0·00032° C. per centimetre. Following Holmes, I shall take m to be 0·00005° C. per centimetre. S is the melting point of average continental rocks. This is somewhat uncertain, since it is affected by the presence of water. It will be assumed to be 1200° C. In the absence of water this would probably be about correct for granitic rocks; it would be higher for the basaltic rocks that occur at a small depth, but water would reduce it, so that this value may not be far wrong on the whole. The conductivity k is about 0·005 C.G.S. for average rocks;

$$\rho = 2{\cdot}8 \text{ g/cm.}^3, \tag{10}$$

$$\sigma = 0{\cdot}25 \text{ cals/g. } 1^\circ \text{ C.} \tag{11}$$

Then
$$h = 0{\cdot}084. \tag{12}$$

The age of the earth since solidification, denoted by t, we may take to be $1{\cdot}6\times10^9$ years or $5{\cdot}0\times10^{16}$ sec. in accordance with Holmes's results, based on

* Holmes (*loc. cit.*).

† This solution has previously been obtained by L. R. Ingersoll and O. J. Zobel, 'Mathematical Theory of Heat Conduction,' Ginn and Co., 1913.

the uranium-lead ratio. A is $1 \cdot 03 \times 10^{-12}$ cal./cm.^{3} sec.* Then in equation (9) all the quantities are known except a, for which we may therefore solve. We find

$$a = 6 \cdot 5 \times 10^{-7}/\text{cm}. \tag{13}$$

Thus at a depth of 15·3 km. the rate of production of heat by radioactivity should be e^{-1} of that at the surface. The ratio of the rates of production of heat by basic and acidic rocks is about 0·29, so that if the rocks at a depth of 10 km. are predominantly basic this result will be reasonable. On geological grounds this appears acceptable. It may be noticed that the value of a found by Holmes was $4 \cdot 0 \times 10^{-7}$/cm.; the difference is chiefly due to his having, by means of a special assumption about sub-oceanic conditions, attempted to deal with average rocks, whereas the data adopted here definitely refer to the continental areas.

We may observe that with the numerical values now obtained,

$$\gamma = 12,$$

approximately; unless x is greater than 2000 km., $\gamma - \lambda$ will be greater than 3, and it will be justifiable to approximate to the error functions according to the formula

$$1 - \text{Erf } \theta = \frac{e^{-\theta^2}}{\theta \sqrt{(\pi)}}. \tag{14}$$

The equation (6) now simplifies to

$$V = mx + \left(S - \frac{A}{a^2 k}\right) \text{Erf } \lambda + \frac{A}{a^2 k}(1 - e^{-ax}) + \frac{A \lambda e^{-\lambda^2}}{a^2 k \sqrt{(\pi)} (\gamma^2 - \lambda^2)}. \tag{15}$$

The last term is at most of order $1/\gamma^2$ of the third, and with the values adopted this ratio is about 0·01. Thus the last term is small compared with the second and third, and the assumption that it is negligible is justified.

The thermal data available for the rocks below the ocean are still more scanty than those for continental rocks, owing to the fact that no deep oceanic boring has yet been made. The value of such a boring in guiding research in this subject could hardly be exaggerated. There is, however, some reason to believe that the surface rocks of the oceans are predominantly basic. It will be assumed here that they have a content of radioactive matter equal to that of basic igneous rocks within the continents, giving a rate of heat production of $0 \cdot 29 \times 12^{-12}$ cal./cm.^{3} sec. The rate of evolution of heat will be supposed to decrease with depth according to the same law as within the continents. With these assumptions the problem of the cooling of the sub-oceanic regions is determinate.

* Holmes, 'Geological Magazine,' February, 1915, pp. 60–71.

2. *The Compression Available for Mountain Building.*

In my former paper I considered the effect of thermal contraction in a spherical earth in producing crumpling of the surface rocks. The formula found was based on an expression for the temperature agreeing with (15) except that the last term was omitted. The error had not then been evaluated, but has now been proved unimportant. The compression was found to be given by

$$\mathrm{K} = \frac{6\beta h}{c}\left(\frac{t}{\pi}\right)^{\frac{1}{2}}\{\epsilon+\epsilon'\alpha+\epsilon' mh\sqrt{(\pi t)}+\epsilon'\beta/\sqrt{(2)}\}, \tag{16}$$

where K is the fraction of its length by which any great circle has been shortened by compression;

$$\alpha = \mathrm{A}/a^2k \qquad \beta = \mathrm{S}-\alpha, \tag{17}$$

c is the radius of the earth;

the coefficient of linear expansion of the rocks is $\epsilon+\epsilon'\mathrm{V}$, where ϵ and ϵ' are constants.

We have

$$c = 6{\cdot}4\times 10^8 \text{ cm.} \tag{18}$$

$$\epsilon = 7\times 10^{-6}\div 1^\circ \text{ C.}, \tag{19}$$

$$\epsilon' = 2{\cdot}4\times 10^{-8}\div(1^\circ \text{ C.})^2, \tag{20}$$

following Fizeau.

For continental regions we have

$$\alpha = 490^\circ; \quad \beta = 710^\circ, \quad \text{and hence} \quad \mathrm{K} = 3{\cdot}7\times 10^{-3}. \tag{21}$$

For oceanic regions,

$$\alpha = 139^\circ; \quad \beta = 1061^\circ, \quad \text{and} \quad \mathrm{K} = 5{\cdot}2\times 10^{-3}. \tag{22}$$

The area of the land surface of the earth is $1{\cdot}45\times 10^{18}$ cm.^{2}, and that of the ocean surface is $3{\cdot}67\times 10^{18}$ cm.^{2}. Hence, remembering that the proportional reduction of an area is double that of a length, we find that the reduction of the earth's surface by crumpling is 49×10^{15} cm.^{2}, of which 11×10^{15} cm.^{2} comes from the land and the rest from the sea. The average reduction in length of a great circle of the earth by compression is found to be 170 km.

In my previous paper the areal compression needed to account for all the existing mountains was estimated to be 19×10^{15} cm.^{2}, so that the amount available appears to be two and a half times that required. The surplus, however, is not so great as this. It might be thought that oceanic crumpling should not be included at all, since it would give rise to mountains on the sea-bottom, and not to the observed continental mountains. This is, however, only partly true. On the present hypothesis the lower radioactivity of sub-oceanic rocks has enabled them to cool to a greater extent than sub-continental rocks. In addition, it appears that basic rocks are on the whole stronger than acidic ones; thus basalt has, under ordinary conditions, a crushing strength of

12×10^8 dynes per square centimetre, as against 8×10^8 dynes per square centimetre for granite.* On both grounds, therefore, the rocks below the oceans must be stronger than those below the continents. Now, where the compressed ocean floor abuts on a compressed continent, the weaker will be the first to give way; the ocean floor will be partly forced over the continent, and the marginal rocks of the continent will be driven inwards and piled up over those already there. This will produce ranges of mountains nearly parallel to the coast, with much overthrusting, which correspond to the Pacific type of mountain range, such as the Rockies and the Andes. Such mountains are evidently, if this theory is correct, formed by the relief of sub-oceanic and not sub-continental compression, so that in these cases the compression produced has been derived from the oceanic rocks. It is not possible to state how much of the compression needed to form other mountains has been provided in a similar way, but the amount may be considerable. We may therefore say that the compression available on the thermal contraction theory is probably enough to account for all the existing mountains of the globe.

It may be pointed out that if the age of the earth since solidification was only $1{\cdot}6 \times 10^7$ years, which is about that determined from the cooling of the sun if no unknown source of energy is available, $S/h\sqrt{(\pi t)}$ is $0{\cdot}00036°$ C. per centimetre. Thus, at the present time, even without any primitive increase of temperature inwards, and without any supply of heat from radioactive sources, the rate of conduction of heat from the earth would exceed the actual rate. This presents a difficulty to this theory, and is avoided by the acceptance of the greater age of the earth.

[*Note added May* 30, 1921.—It is often stated as an objection to the thermal contraction theory of mountain building that the gradual cooling would give rise to mountains in all geological ages, whereas it is known that there have been long intervals of quiescence. When the strength of the earth's crust is taken into account, however, these intervals of quiescence become a natural consequence of the theory. It appears that the stress-difference produced in the crust by the cooling of the interior increases only gradually, and it is not until it reaches the crushing strength of the rocks that yield takes place and mountain building follows. A local failure of strength will rapidly spread and the formation of mountains will continue until the stresses are almost completely relieved. Thus the fact that all the great mountain chains were elevated in Tertiary times is readily accounted for.

In the case of Palæozoic and pre-Palæozoic mountains, many possible

* Landolt and Börnstein, 'Physikalische Tabelle.'

causes of uplift can be suggested, as Dr. J. W. Evans has pointed out. If tidal friction for the last 1000 million years has been as potent as now (and it is more likely to have been more potent than less), the earth would at the beginning of that interval have rotated twice as fast as now, and the corresponding change of ellipticity must have been of great importance in mountain building.]

3. *The Compensation of Crustal Inequalities.*

The arguments concerning the existence of some horizontal variation of density within the crust of the earth, which partially neutralises the disturbances of surface gravity caused by the irregular form of the land surface, are partly geodetic and partly physical; but the latter have scarcely been adequately discussed. It was observed by Bouguer in Peru that the attraction of a mountain there was much less than would be expected from its size, and Laplace inferred from this that the matter composing it was lighter than the average. Pratt made a similar discovery about the Himalayas, and from his work the modern developments of the theory have sprung. The essential feature of the theory of isostasy is that the matter below mountains is supposed to be lighter than the average, while that below seas is, on the whole, denser. The amount of the excess or deficiency is just enough to balance the attraction of the surface inequalities on a particle a considerable distance away. It follows that, if an equipotential surface be taken below these inequalities, and columns of the same cross-section bored down to it from the surface, the masses in all of them are equal. This may be expressed more precisely by saying that the geoid has the form of the outer surface of a fluid earth, with the same radial density-distribution and speed of rotation as the actual earth. Now we come to the physical aspect of the problem. This result would hold if the pressure all over the standard equipotential were nearly uniform, and the interior of the earth nearly free from shearing stress. This condition indicates that, within the standard equipotential, the stress across any plane is probably perpendicular to that plane; in other words, the interior is in a hydrostatic state.

There are two alternative reasons why this hydrostatic state should exist. It may be primitive; that is to say, the inequalities may have been compensated from the earliest times. The question of its origin will then be bound up with that of the inequalities themselves. Though this may be true of part of the compensation, it is certainly not true in every case of compensation. For though it is usually held that the great continents and oceans have, on the whole, kept very much the same positions throughout

geological time, it is quite certain that there have been very great changes in level in detail; in particular, cases of rising and sinking of the coast are well known, and the elevation of mountain ranges must have produced huge stresses. It is concerning these very features that the geodetic evidence for the existence of some sort of compensation is clearest, as may be seen from the results of the United States Geodetic Survey and the Indian Survey. There must therefore be some agency that tends towards the readjustment of departures from the hydrostatic state, possibly coarsely and intermittently, but yet ensuring that no departure exceeding a certain amount, in the United States apparently about 200 metres, can persist for a long time. It may even be that many surface inequalities are compensated from the time they are formed. The nature of this readjustment requires some consideration. The assumption of Hayford and other American geodesists is that, below any mountain, the density is below the average by an amount proportional to the height of the mountain, this reduction extending to a uniform depth of 96 km., according to the latest calculations of William Bowie.* In previous investigations, it was found that the assumption that the defect of density is the same at all depths down to this level of compensation, makes little difference to the accuracy with which the solution obtainable represents the observed gravity anomalies. Now it is obvious that if a mountain were the result of addition of matter to, or a denudation valley the result of removal of matter from, the outer layers of the earth, any compensation produced by expansion or contraction below must arise from displacements taking place both horizontally as well as vertically. For, if there is no horizontal motion, the mass in a vertical column must remain the same, and there can therefore be no compensation. Thus horizontal displacement is essential to the production of compensation.

Again, the nature of the compensation must not violate the known properties of matter. Ordinary matter does not decrease in density when a pressure is applied to it, as is supposed to happen in this case when the mountain is superposed. The requisite reduction in mass cannot therefore be caused by the mere mechanical effects of pressure on a single substance; if it is an effect of pressure at all, it must consist in the partial replacement of a dense substance by a light one, or *vice versâ.* Accordingly, the mechanism of the adjustment must be, in the case of the mountain, that heavy matter flows out horizontally from below, while a certain amount of lighter matter may enter in the upper layers, though there is no clear evidence on this point. The matter near the bounding surface between the

* 'Investigations of Gravity and Isostasy,' Spec. Pub. No. 40, U.S. Coast and Geodetic Survey, 1917.

light and heavy materials must sink downwards to some extent, so as to continue to fit the zone whence the heavy matter has flowed; but, as the rocks of which the mountain is composed are lighter than those below, compensation will be reached when the volume of the rocks displaced is less than that of the mountain, so that the mountain will remain as a projection on the surface even when the process is complete. In the same way, the removal of matter by denudation and its re-deposition elsewhere give rise to inequalities, which will in time come to be compensated. The flow must evidently take place below the layer of compensation, as was pointed out by Barrell;* for the layer in which it occurs must be in a hydrostatic state, and therefore can transmit no shearing stress; on the other hand, any horizontal displacement can only be produced by shearing stress; and hence a surface inequality can never cause a horizontal displacement on the opposite side of a layer that is by hypothesis unable to transmit shearing stress. Hayford's hypothesis, however, demands that an inequality shall cause a displacement as far down as the level of compensation; it follows that the flow must take place below, and probably a long way below, the layer of compensation.

Suppose then that the density of the matter constituting the mountain is σ, and that the mean elevation of its surface over the mean surface of the land would, in the absence of compensation, be h. Let the density of the matter at the level of the flow be ρ_0. Then compensation will be attained if the thickness of the latter is diminished by $\sigma h/\rho_0$. The region above this will sink by approximately this distance throughout. Now if the initial density at distance r from the centre was ρ, this shows that the density at the same distance afterwards is equal to the original density at distance $r+\sigma h/\rho_0$ from the centre, or $\rho+\frac{\sigma h}{\rho_0}\frac{d\rho}{dr}$. The density diminishes towards the surface, so that the change in density owing to the compensation, being $\frac{\sigma h}{\rho_0}\frac{d\rho}{dr}$, is negative. This is a constant if ρ is a linear function of r. Accordingly, Hayford's hypothesis that the defect of density is the same at all levels is true if the density under plains increases uniformly with depth until we reach the layer of compensation, below which it remains constant as far down as the layer of flow. There is nothing inherently improbable about this, and it is clear that the charges of artificiality often made against Hayford's hypothesis are unfounded. If there is a sudden change in density at any level, the compensation must be concentrated in that level. There is some evidence, from the behaviour of earthquake waves, that there is a discontinuity of substance at a depth of about 30 km., while the density must

* 'Journal of Geology,' 1915–16.

increase gradually with depth on account of the increasing percentage of heavy constituents. Thus the true distribution of compensation is probably intermediate between these two extreme hypotheses.

It must be noticed, however, that the American observations deal almost wholly with the compensation of differences of level within a continent. The smallness of the residuals given by Hayford's Solution C,* in which the sea was supposed compensated according to Helmert's hypothesis and the land uncompensated, in comparison with the Bouguer solution, in which both were supposed uncompensated, shows that the general features of the ocean are probably compensated, at least within some distance of the order of 2000 km. of the coast. It affords no reason for believing that even the major inequalities are compensated at greater distances from land than this, and none for believing that inequalities of small horizontal extent are compensated even within the region for which it really provides information.

Accordingly, the problem of the compensation of the oceans cannot be regarded as solved. Two important questions remain: first, whether the oceanic regions as a whole are associated with rocks of higher density; and second, whether regions of specially great or small depth in the oceans, with horizontal dimensions small compared with the oceans as a whole, are associated with especially large or small quantities of dense rock so as to compensate for their departures from the mean depth. The first question will be referred to as that of the general compensation of the oceans, and the second as that of the local compensation of the oceans. In the following sections I propose to deal with certain physical arguments that appear to throw light upon these matters, pending a decision from the determination of gravity at sea, and to develop further consequences of the same theory.

4. *The General Compensation of the Oceans.*

Apart from the ellipticity of the earth due to rotation, which is practically the same for land and sea, so that there is no inequality in this case sufficiently important for the question of its compensation to arise, the chief departure of the earth from the spherical form is represented by the first harmonic. This expresses the existence of the land and water hemispheres, the former of which includes nearly all the land of the earth. It is not difficult to see that any first harmonic in the difference of elevation between the lithosphere and the geoid must be compensated. In a homogeneous earth such a first harmonic inequality could not exist. For it would be equivalent to a mere bodily displacement, the earth remaining a perfect spheroid. Hence the equipotential surfaces would be displaced by the same

* 'The Figure of the Earth and Isostasy,' 1909, p. 169.

amount, so that their centre would again be at the new position of the centre of the earth. Thus the geoid and the lithosphere would be concentric, and the average depth of the ocean in any hemisphere would be the same.

When heterogeneity of density is taken into account a peculiar result is obtained. It is well known that the potential due to a gravitating body at a distant point is of the form

$$f\,(\mathrm{M}/r + \text{solid harmonics of the second and higher orders}), \qquad (23)$$

where f is the constant of gravitation, M the total mass, and r the distance of the point from the centre of mass of the body. In the case of bodies differing only slightly from the spherical form, this formula is known to be accurate to the first order of magnitude right up to the surface. Since for purely first harmonic deformations no harmonics of higher order than the first can arise, the equipotentials right up to the surface are spheres whose centre is the centre of mass of the body. In other words, whatever first harmonic deformations may exist, they produce equal displacements of the centre of mass of the whole and of the equipotentials near the surface. Thus, whatever may be the physical conditions that may give rise to a first harmonic deformation, it is necessarily completely compensated, merely as a consequence of the law of gravitation itself. No information about the nature of isostasy, therefore, can be obtained by considering the inequality that gives rise to the land and water hemispheres; it would be compensated even if the earth were a perfectly rigid body. One observable consequence of this must be that gravity far from shore in the Pacific Ocean cannot show a systematic difference from its values at geoid level in the continents.

At the same time, no light can be thrown on the general question of the compensation of the oceans by considering the land and water hemispheres; it must be approached either by means of special information or from physical considerations about the earth's interior. Barrell considered that the American and other observations he discussed agreed best with the supposition that the layers in which isostatic flow takes place, which he called the asthenosphere, are at a depth of the order of 300 km. This estimate agrees significantly with what would be expected on the theory of a cooling earth. We may legitimately suppose that at least the siliceous part of the earth, extending to a depth of the order of 1200 km., was fluid when solidification was taking place at the surface or near it. The melting point in the interior would of course be much raised by pressure, but it appears likely that the convective agitation that must have taken place during solidification, owing to the sinking and re-melting of surface fragments, would ensure that a very great

depth of rock solidified at about the same time. Since then, however, the interior has scarcely cooled at all, and must therefore still be in very nearly the same condition as immediately after solidification. We should, therefore, expect it to be extremely ready to flow under small shearing stress lasting for a considerable time. The fact that earthquake waves can be transmitted through it does not affect this argument, since the stresses involved in them continue for only a few seconds. Cooling near the surface, on the other hand, has very much increased the strength there. Thus we should expect that the depth where strength becomes low should be of the same order of magnitude as the depth at which cooling is just becoming important. This can be seen from (15) to be of the order of $2ht^{\frac{1}{2}}$ or 370 km., in good agreement with Barrell's estimate, again on the hypothesis of the longer age of the earth.

This estimate of $2ht^{\frac{1}{2}}$ is equally applicable to continental and oceanic regions, and consequently we may infer that any inequality capable of producing appreciable shearing stresses at a depth of about 400 km. must cause flow. Thus any inequality of height more than 200 m. and extent more than 1000 km. should be compensated. The general compensation of the oceans may therefore be inferred from the theory.

The local compensation of the oceans is on a somewhat different footing. At depths of the order of 200 km. the cooling has been greater under the oceans than under the continents, and accordingly it is likely that the size of the uncompensated inequalities needed to cause flow there is greater than that indicated by Hayford's work for similar inequalities in the United States. Thus larger uncompensated inequalities, of extent comparable with 600 km., may be expected to exist. These would produce maximum stress-differences at depths of about 200 km., and at greater depths the stress-differences would rapidly decline, so that their existence would be compatible with the much lower strength of the zone below.

5. *On the Origin of Oceanic Deeps.*

In consequence of the greater cooling of the rocks at depths of 100 to 200 km. below the oceans, these must have tended to contract more than rocks at equal depths below the continents. The surface, however, must have remained at almost constant temperature on account of the cold water at the sea bottom. This would give rise to a phenomenon comparable with the bending of the covers of a book when held in front of a fire. The contraction of the side next the fire is due to loss of moisture, in the latter case, and to fall of temperature in the former. The margins of the sub-oceanic crust must tend to curl towards the contracting side; thus the ocean floor must

tend to sink near the coasts and to rise in the middle. This will produce an additional downward pressure near the margins and a reduced one in the middle, which must cause a stress-difference in the asthenosphere. This will therefore yield so as to permit such a crust movement to take place.

At a depth of the order of 100 km. oceanic rocks have cooled about 300° more than continental ones. Young's modulus for materials at this depth is known from earthquake data to be about 15×10^9 dynes/cm.^{2}, and hence the tension needed to prevent the crust from curling must be about 3×10^9 dynes/cm.^{2}, with the coefficient of linear expansion already adopted. This is a very large amount. The crushing strength of basalt, which is probably typical of these deep rocks, is $1{\cdot}2 \times 10^9$ dynes/cm.^{2}. It must be remembered, however, that the pressure is very great, while the temperature is on an average several hundred degrees below the melting point. In these circumstances it is known from experiment that the strengths of rocks are many times greater than under atmospheric pressure. It is therefore probable that the rocks could endure this stress-difference without yielding, even if the ends were held tight so as to prevent the curvature needed to readjust it.

The determination of a complete formal solution of the problem of the straining of a portion of the earth's crust by cooling would be very laborious. As only an estimate of the order of magnitude of the possible deformation is required, we shall, therefore, discuss only the following related problem for the cylindrical case.

A fluid is in equilibrium in the form of an infinite circular cylinder, the force of gravity at the surface being normal to the axis. On its surface an infinite strip of solid matter floats, its curvature being equal to that of the surface of the fluid. It then contracts by cooling, the relative contraction that would be produced at any depth, x, in the absence of stress, being α. The solid is supposed to have Young's modulus E, and Poisson's ratio zero. It is assumed that after contraction it remains a portion of a cylinder. Find the elevation of the centre above the margins.

Let 2R be the width of the strip, c the radius of the cylinder, and c_1 that of the nearest cylindrical strip after the cooling. Let the elevation of any point of the surface above the undisturbed position be ζ, and the difference between the values of ζ at the centre and at the margins, H. Then the original length of the arc at depth x was $2\mathrm{R}\left(1-\frac{x}{c}\right)$, and the natural length after cooling is $2\mathrm{R}\left(1-\frac{x}{c}-\alpha\right)$. The actual length is $2\mathrm{R}\left(1-\frac{x}{c_1}\right)$. Hence the amount of stretching is $2\mathrm{R}\left(\frac{x}{c}+\alpha-\frac{x}{c_1}\right)$, and the tension is $\mathrm{E}\left(\frac{x}{c}+\alpha-\frac{x}{c_1}\right)$. The

strain energy is therefore $\mathrm{ER}\int\left(\frac{x}{c}-\frac{x}{c_1}+\alpha\right)^2 dx$ taken through the strip. The gravitational energy is $\int \frac{1}{2}g\rho\zeta^2 dy$, where y is the element of arc of a section of the cylinder, and the integral is taken across the strip. The approximate solution desired will be obtained by making the total energy a minimum.

Remembering that the mean value of ζ must be zero, from the condition that the strip is still floating, we see that

$$\zeta = \left(\frac{1}{3}-\frac{y^2}{\mathrm{R}^2}\right)\mathrm{H}. \tag{24}$$

We also notice that

$$c_1\left(1-\cos\frac{\mathrm{R}}{c_1}\right) = c\left(1-\cos\frac{\mathrm{R}}{c}\right)+\mathrm{H}. \tag{25}$$

If powers of R above the second be omitted, this gives

$$1/c_1-1/c = 2\mathrm{H}/\mathrm{R}^2. \tag{26}$$

The energy reduces to

$$\frac{4}{45}g\rho\mathrm{H}^2\mathrm{R}+\mathrm{ER}\int\left(\alpha-\frac{2\mathrm{H}x}{\mathrm{R}^2}\right)^2 dx. \tag{27}$$

The equilibrium condition is obtained by differentiating this with regard to H. Hence

$$\frac{8}{45}g\rho\mathrm{H}-\mathrm{E}\int\frac{4x}{\mathrm{R}^2}\left(\alpha-\frac{2\mathrm{H}x}{\mathrm{R}^2}\right)dx = 0. \tag{28}$$

With $g = 981$ cm./sec.^{2},

$$\alpha = 4\times10^{-3}, \qquad x = 100 \text{ km.}, \qquad \mathrm{R} = 2000 \text{ km.}$$

we see that the first term is of the order of sixty times that under the integral sign involving H. In other words, the tendency to curve is mostly balanced by the disturbance of hydrostatic pressure, so that the curvature that actually takes place is only a small fraction of what would occur in the absence of gravity. With the above values of the quantities involved, H is found to be nearly 1 km. Thus, we should expect to find that the chief oceans will have regions around their margins deeper, by a distance of the order of a kilometre, than the centres.

This bears a suggestive resemblance to the facts with regard to oceanic deeps. All the chief deeps in the Pacific are near the margin: there are depths of 8500 m. near the Kurile Islands, 6000 m. off the Aleutian Islands, 7600 m. off Chile, 9000 m. near the Marianne Islands, and a strip of depth 9000 m. to the north of New Zealand. The last two are now in mid-ocean, but are near the edge of a former continent. The mean depth of the oceans is about 5000 m., so that some of these regions are too deep to be altogether accounted for in this way unless the difference of cooling is supposed to

extend to a greater depth than 100 km.: but this, of course, is quite probable. The same is true on a smaller scale in the Indian Ocean. Off the coasts of Australia, Java and Sumatra there are depths of 6000 m., while the middle is occupied by a huge area whose depth does not exceed 4000 m. In the Atlantic, again, shallow strips extend from the south up the middle, past S. Georgia, Tristan d'Acunha, St. Helena, and Ascension, and from the north right down the centre to the Azores. Thus, each ocean has a region of smaller depth in the middle, as is predicted by the theory.

A possible test of this theory is afforded by the fact that the inflow of matter from the margins towards the centre required by the theory is not caused by any additional weight on the surface. Thus, it corresponds to a net transference of mass towards a particular region. The mass per unit area at a deep should therefore be less than that at the centre of the ocean. Hence this should be indicated by a true defect of gravity at sea-level in the deeps; in a gravity determination it should therefore appear as if the deeps were uncompensated. The evidence at present available on this point is meagre. Duffield's observations* on the "Morea" indicate a defect of gravity over the deep parts of the Indian Ocean of about the theoretical amount, but are open to some uncertainty. Nevertheless it would be a remarkable coincidence if accidental errors have happened to produce low values of gravity at all the places where they should theoretically have been expected, and so far as the observations go they definitely support the theory.

The gravity anomalies here predicted do not arise from a permanent strength in the asthenosphere, which is on this theory in a hydrostatic state, as under a mountain chain; they arise from the strength and tendency to curvature of the strong upper crust. Faulting may occur near the margins. A reversed effect may be looked for in the continents, but is probably masked by denudation and sedimentary rocks.

6. *The Formation of Geosynclines.*

It has been seen that the addition of a thickness h of matter of density σ depresses the crust by an amount $\sigma h/\rho_0$, so that the upper surface of the new matter is $\left(1-\frac{\sigma}{\rho_0}\right)h$ above the original surface. If the deposition takes place from water, the additional mass is only the excess of the mass of the sediments over that of the water displaced. Let ρ_1 be the density of the sediments and ρ_2 that of the water, h the depth of sediment deposited, and x the depression of the original surface. Then the mass per unit area is

* "On the Determination of Gravity at Sea," 'Brit. Assoc. Report, Newcastle,' 1916, pp. 549–565.

increased by $\rho_1 h - (h-x)\rho_2 - \rho_0 x$, and the condition for compensation is that this shall vanish. We find that

$$(h-x)/h = (\rho_0 - \rho_1)/(\rho_0 - \rho_2). \tag{29}$$

But $h-x$ is the reduction in the depth of the water. We therefore see that the depth of sediment that can be deposited in a sea whose initial depth is known is a determinate multiple of this depth. If we take ρ_0 to be 3·2, which is probably typical of the rocks at a depth of some hundreds of kilometres, ρ_1 to be 2·2, and ρ_2 to be 1·0, it is seen that the maximum depth of the sediments is 2·2 times the original depth of the water.

The possibility of deposition of sediments to a depth far greater than the initial depth of the water in which they are formed is obviously of considerable geological importance; but the depression of the sea bottom that can be produced in this way has been much exaggerated, as has been pointed out by A. Morley Davies.* If the stress differences present become too small to cause flow, compensation will not proceed. Accordingly, the adjustment that takes place can never be greater, and may be less, than the amount needed to give compensation. But very many cases are known where the existing deposits are far more than 2·2 times as deep as the sea can have been when their formation commenced, and for these the theory of isostatic compensation is therefore inadequate. To account for them we must have a theory that will explain how the crust in a region of deposition can be depressed by an amount far in excess of that needed to give compensation. Again, some of these sedimented regions afterwards rise far above sea-level, implying a flow of matter into them for which there could be no explanation if the depression of the crust at the end of the sedimentation was less than or equal to the amount needed to neutralise the effect of the weight of the sediments. Accordingly, there must be important departure from isostasy at certain stages of the development of such regions, and no explanation of the existence of sedimental rocks above sea-level can be satisfactory unless it takes them into account.

A method by which these extensive sedimented regions can afterwards be uplifted is suggested by the theory of the origin of deeps just described. The sediments from a continent must often be deposited in a gradually developing deep. Their weight will accentuate the tendency already existing for that region to sink. Hence the stresses in the crust will be increased, and may lead to fracture. When this takes place, the crust on the oceanward side will be free to bend down further. That on the landward side of the fault, however, will now have nothing to hold it down

* 'Geological Magazine,' 1918, pp. 125 and 233; E. M. Anderson, *loc. cit.*, p. 192.

except the weight of the sediments. Accordingly, it will be free to rise above sea-level. If compression occurs afterwards, a greater thickness of light sedimentary rocks is accumulated, and hence the surface will be raised still higher.

7. *The Stresses in an Elastic Solid due to Changes of Temperature.*

In the present discussion the earth will be replaced by an elastic sphere of variable elasticity and density, and the changes of temperature considered will always be symmetrical about the centre and confined to the neighbourhood of the surface. The errors arising from the neglect of the ellipticity of the earth are very small.

Let the excess of temperature at any point (x, y, z) over the initial temperature there be V; (u, v, w) the component displacements of the matter originally there; n the coefficient of linear expansion; λ and μ the elastic coefficients of the matter there; and P, Q, R, S, T, U the six stress-components.

Then the equations of equilibrium are three of the form

$$\frac{\partial \mathrm{P}}{\partial x}+\frac{\partial \mathrm{U}}{\partial y}+\frac{\partial \mathrm{T}}{\partial z}+\rho \mathrm{X} = 0, \tag{30}$$

where ρ is the density and (X, Y, Z) the bodily force per unit mass. Now consider how the stresses P, Q, R, S, T, U can arise. Before the change of temperature they had the values P_0, Q_0, R_0, S_0, T_0, U_0, say. When an element moves from x, y, z to $x+u$, $y+v$, $z+w$, it retains its original stress, to which is added the new part, namely, the stress needed to produce the changes of the element in size and shape, from the form it would have had if it simply expanded without strain, to the form it actually takes. The alterations in dimensions due to simple expansion in the absence of stress would make

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}=\frac{\partial w}{\partial z}= n\mathrm{V}\,; \qquad \frac{\partial u}{\partial y}=\frac{\partial u}{\partial z}=\frac{\partial v}{\partial x}=\frac{\partial v}{\partial z}=\frac{\partial w}{\partial x}=\frac{\partial w}{\partial y}=0. \tag{31}$$

Now the six components of stress depend on the changes of the quantities

$$\frac{\partial u}{\partial x},\ \frac{\partial v}{\partial y},\ \frac{\partial w}{\partial z},\ \frac{\partial w}{\partial y}+\frac{\partial v}{\partial z},\ \frac{\partial u}{\partial z}+\frac{\partial w}{\partial x},\ \frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}, \tag{32}$$

from their amounts corresponding to simple expansion to their final amounts. Thus, in the stress-strain relations of the ordinary theory of the deformation of an elastic solid we must put the first three of the strains equal to $\frac{\partial u}{\partial x}-n\mathrm{V}$, $\frac{\partial v}{\partial y}-n\mathrm{V}$, $\frac{\partial w}{\partial z}-n\mathrm{V}$, while the other three are of the form $\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}$

as in isothermal straining. If the extra stresses be denoted by P_1, etc., their values are therefore given by

$$P_1 = \lambda\delta + 2\mu\frac{\partial u}{\partial x} - (3\lambda+2\mu)\,n\mathrm{V}, \text{ with two symmetrical relations}; \quad (33)$$

$$S_1 = \mu\left(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right) \text{ with two symmetrical relations; where} \quad (34)$$

$$\delta = \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}. \quad (35)$$

The bodily forces X, Y, Z can also be regarded as composed of two parts, namely, X_0, Y_0, Z_0, the forces at x, y, z before the deformation, and X_1, Y_1, Z_1, the additional forces acting on the same particle caused by the deformation. The equations of strain are then three, of the form

$$\frac{\partial P_0}{\partial x}+\frac{\partial U_0}{\partial y}+\frac{\partial T_0}{\partial z}+\frac{\partial}{\partial x}\left(\lambda\delta+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left\{\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right\}+\frac{\partial}{\partial z}\left\{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right\}$$
$$-\frac{\partial}{\partial x}\{(3\lambda+2\mu)\,n\mathrm{V}\}+\rho\,(X_0+X_1) = 0. \quad (36)$$

Further progress is impossible without knowledge of the initial stress. If this be assumed to be hydrostatic pressure, as would be correct in a planet which had solidified while stirred up by bodily convection currents produced by surface cooling, we have

$$S_0 = T_0 = U_0 = 0, \qquad P_0 = Q_0 = R_0, \quad (37)$$

$$\frac{\partial P_0}{\partial x}+\rho_0 X_0 = 0, \text{ etc.} \quad (38)$$

We have also

$$\rho = \rho_0 - \frac{\partial}{\partial x}(\rho_0 u) - \frac{\partial}{\partial y}(\rho_0 v) - \frac{\partial}{\partial z}(\rho_0 w) = \rho_0+\rho_1, \text{ say}. \quad (39)$$

Put
$$(3\lambda+2\mu)\,n\mathrm{V} = \gamma. \quad (40)$$

The equations of equilibrium therefore become

$$\frac{\partial}{\partial x}\left(\lambda\delta+2\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left\{\mu\left(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right)\right\}+\frac{\partial}{\partial z}\left\{\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)\right\}-\frac{\partial\gamma}{\partial x}$$
$$+\rho_0 X_1+\rho_1 X_0 = 0, \quad (41)$$

with two similar equations. These are perfectly general, provided the squares of the displacements can be neglected, and will still hold in a heterogeneous and non-spherical body.*

* *Cf.* Love, 'Problems of Geodynamics,' Camb. Univ. Press, 1911, pp. 89–92.

8. *Application to the Earth.*

Let the radial displacement at distance r from the centre be q. Then by symmetry, the origin being at the centre, we have

$$u = qx/r, \qquad v = qy/r, \qquad w = qz/r. \tag{42}$$

The radial force acting on unit mass is g, and is of course in general a function of r. Thus

$$X_0 = -gx/r, \text{ etc.} \tag{43}$$

The force acting on unit mass after the displacement is $X_0\left(\frac{r-q}{r}\right)^2$, so that

$$X_1 = 2qgx/r^2. \tag{44}$$

Also

$$\delta = \frac{1}{r^2}\frac{d}{dr}(r^2 q), \tag{45}$$

$$\rho_1 = -\rho_0\delta - q\frac{d\rho_0}{dr}. \tag{46}$$

Substituting in (41), and remembering that λ and μ are functions of r alone, we find that all three equations are satisfied if

$$\frac{d}{dr}\{(\lambda+2\mu)\delta\} - 2q\left(\frac{\mu}{r^2}+\frac{2}{r}\frac{d\mu}{dr}\right) - \frac{\partial\gamma}{\partial r} + g\rho_0\left(\frac{2q}{r}+\delta\right) + gq\frac{d\rho_0}{dr} = 0. \tag{47}$$

If, now, H be of the order of magnitude of the depth to which cooling extends, we see that dq/dr must be of the order q/H, and therefore large compared with q/r in general. If c be the radius of the earth, we see that the first term in the last equation is of order $\lambda q/\text{H}^2$, the second $\lambda q/c^2$, the fourth $g\rho_0 q/\text{H}$, and the last $g\rho_0 q/c$. Accordingly, the fourth term is about a tenth of the first, and the others, with the exception of the third, not more than a hundredth of it. We can accordingly reduce the equation to

$$\frac{d}{dr}\{(\lambda+2\mu)\delta\} + g\rho_0\delta - \frac{d\gamma}{dr} = 0. \tag{48}$$

Hence

$$(\lambda+2\mu)\delta = \exp\left\{-\int^r \frac{g\rho_0}{\lambda+2\mu}dr\right\}\int^r\left[\exp\int^r\frac{g\rho_0}{\lambda+2\mu}dr\right]\frac{d\gamma}{dr}dr, \tag{49}$$

which becomes, if we neglect the square of $g\rho_0\text{H}/\lambda$,

$$= \gamma - \int^r \frac{g\rho_0\gamma}{\lambda+2\mu}dr. \tag{50}$$

The lower limit of the integral is at present left undetermined.

If the point considered be on the axis of x, the additional stresses are at once found to be given by

$$P_1 = \lambda\delta + 2\mu\, dq/dr - \gamma, \tag{51}$$

$$Q_1 = R_1 = \lambda\delta + 2\mu\, q/r - \gamma, \tag{52}$$

$$S_1 = T_1 = U_1 = 0. \tag{53}$$

Thus the principal stresses are radial and tangential, and it is at once seen from symmetry that this must be true at all points. Now, the tendency of the matter to flow or rupture is determined by the difference between the greatest and least of the principal stresses. As the initial stresses were equal in all directions, they do not affect the question. If P, the radial stress, be algebraically the greater, horizontal fracture will tend to occur; and if Q be the greater, the fractures will be vertical. Now,

$$\mathrm{P}-\mathrm{Q} = 2\mu\left(\frac{dq}{dr}-\frac{q}{r}\right) = 2\mu\left(\delta-\frac{3}{r^2}\int_0^r r^2\delta\,dr\right)$$

$$= \frac{2\mu}{r^3}\int_0^r r^3\frac{d\delta}{dr}\,dr = \frac{2\mu}{r^3}\int_0^r \frac{r^3}{\lambda+2\mu}\left(\frac{d\gamma}{dr}-\frac{g\rho_0\gamma}{\lambda+2\mu}\right)dr. \qquad (54)$$

Now let us suppose that no fracture has taken place since solidification, and consider the stress-difference acquired since then. The change of temperature at the centre is insignificant, while at all levels where it is appreciable it is negative. Accordingly, γ is always negative. The first part of the integral is therefore essentially negative, and as the second part is only a small correction to it, the integral as a whole must be negative. Thus the immediate effect of the cooling of the earth is to produce a strong tendency to vertical fracture at all depths.

On the other hand, suppose that the crust has by flow or fracture adjusted itself since solidification until the horizontal tension first produced has been completely relieved. Then the fall of temperature at the surface after this is zero, on the supposition that the temperature there is maintained only by the radiation from the sun, which is supposed constant. Hence γ is zero when $r = 0$, falls to a maximum negative value somewhere near the surface, and then increases again, reaching zero again at the surface. Now, near the surface not only is r greater than at great depths, but, in consequence of the recent work of Knott,* we know that $\lambda+2\mu$ is least near the surface. Hence in the integral $d\gamma/dr$ is multiplied by a larger quantity when it is positive than when it is negative. The first part of the integral is therefore positive when $r = c$, and of order $c^2\mathrm{H}\gamma/(\lambda+2\mu)$. The integrand in the second part of the integral, allowing for the negative sign before it, is always positive. The second part of the integral is therefore positive and of order $g\rho\gamma\mathrm{H}c^3/(\lambda+2\mu)^2$. Now $\{\lambda+2\mu/\rho\}^{\frac{1}{2}}$ is about 7 km.-sec., and therefore the two parts of the integral are seen to be both positive when $r = c$, and of the same order of magnitude. At depths comparable with the level of greatest cooling the integral is negative, as before. Hence, if there has been no variation in the surface

* "The Propagation of Earthquake Waves through the Earth," 'Roy. Soc. Proc. Edin.,' vol. 39, pp. 157–208 (1919).

temperature since hydrostatic conditions were last attained, symmetrical cooling must necessarily lead to a horizontal compression at the surface and a tension below.

9. *The Effects of the Initial Tension near the Surface.*

It has just been shown that the cooling that immediately followed solidification must have produced a tremendous tension in the uppermost layers of the crust. This tension would be practically that which would be developed if a rock cooled down from near its melting point to ordinary temperatures while its ends were kept immovable. No rock could stand such a tension, and, accordingly, it must soon have been relieved in some way. The mode of relief seems fairly clear, though there has been some disagreement about it among previous writers. Its nature and effects appear to have been consistently neglected by geologists, presumably because it took place before the oldest known rocks were formed, but, nevertheless, it is likely to have played a very important part in determining the present configuration of the earth's surface; and relief of tension in modified form has probably continued to produce notable effects even up to the present day. Sir G. H. Darwin, in his investigation of the amount of mountain building to be expected on the Kelvin theory of the cooling of the earth, seems to have thought that the relief would take place by horizontal flow, the surface layers merely becoming somewhat thinner without change of length, and thereby acquiring a new unstressed state. This may be a satisfactory description of the phenomena at great depths, where the pressure is great; but at the surface a rock under tension would break at right angles to the tension just as any rope or bar does in air. Accordingly, the surface must have become honeycombed with vertical cracks. The depths of these would initially be very small and nearly equal, but the differences in depth would gradually grow. For suppose that a crack, A, is slightly deeper than a neighbouring one, B. Then further cooling below will produce a new tension, and the crust at A will have been more weakened by the deeper crack there, and therefore the crack A will commence to grow downwards sooner and more rapidly than the other. When cooling has progressed a long way down, the cracks must become very unequal in depth, and only a few of those originally formed will then be deep enough to continue their growth.

Now it must be remembered that, before solidification, the temperature was not uniform, because the melting point would be raised by pressure, and would therefore rise with the depth. Hence a crack extending downwards must be penetrating regions of higher and higher initial temperature as it

proceeds; but its internal pressure is necessarily atmospheric and practically constant. Hence, although the rocks at any depth are necessarily below their melting points at the pressure normally appropriate to that depth, as soon as they are reached by a crack, there will be a fall of pressure which may lower the melting point sufficiently to cause fusion; all that is needed is that the crack may reach a depth where the actual temperature is as high as the initial temperature at the surface. It may be easily shown from the equation (54) that this will be achieved at the depth of most rapid cooling, which is also a region of great tension, when cooling has proceeded for an interval of the order of 10^7 years, when the depth of the cracks would be comparable with 8 km. When this happens fusion must take place, and magma will be forced up the crack by hydrostatic pressure until the horizontal uniformity of pressure is nearly restored. Now the density of the matter there was probably not very different from that at the surface, and the semi-fluid magma may even have been lighter than the solids at the surface. Hence hydrostatic conditions would not be restored till the crack was practically full. Thus intrusions, not unlike the dykes of the present day, would be formed. Known dykes are not, of course, original examples of this process, being of much later date; all sign of these primitive dykes must have been buried beneath sediments and igneous outpourings long ago. On the moon, however, denudation and sedimentation do not exist, and there some relics may be sought. The well-known rills are not instances, being of later date, as is seen from the fact that in some cases they have broken through crater walls. The radiating streaks are much more likely to afford examples. These are narrow streaks, radiating as a rule from large craters; that they are filled up to the level of the surface is plain from the fact that they are extremely difficult to see under oblique illumination, which would not be the case if there were any difference of level that could cause a shadow to be thrown. In fact, the agreement in level is surprisingly good, considering that it can only arise from a more or less accidental numerical coincidence between the average density of the rocks down to the bottom of the crack, and the density of the rocks at the bottom when fused. The fact that the theory calls for such a coincidence does not, however, afford any argument against it, for the extreme smoothness of the surface of these streaks shows that they must have been filled with a semi-fluid at one time, and the support of this would have to be explained by some such balance whatever theory we should choose to adopt as an explanation of their origin. So far, therefore, we may hold that the theory is confirmed by the existence of these streaks on the moon. As it depends partly on the increase of pressure with depth,

which would be greater in the earth on account of the greater value of gravity, we may have some confidence in its applicability to the earth.

When the lateral ends of cracks are near together, the short length of crust between them has to support the whole of the unrelieved tension, and is therefore a particularly likely part of the crust for the next fracture to occur. Cracks will therefore tend to grow together, and thus will tend to develop into closed polygonal systems. When this takes place, a qualitatively different stage of the process commences. Each polygon is separate from the rest of the crust, and will therefore proceed to develop on its own account. Its surface has long ceased to change in temperature, but cooling continues below, so that there is a tendency to contract underneath. This would tend to close the cracks above by shortening the crust, were it not that the cracks have been closed already by the injected magma. Hence the shortening below can be achieved only by curvature of the crust. The centre of the polygon must therefore rise in the middle relative to the edges; its centre of gravity cannot rise, however, since that would imply the existence of a great additional pressure over the surface, which the weak matter just below would be unable to support. Accordingly, while the centre must rise, the boundary must sink. The matter below will offer little resistance to this depression, but rather will make way for some of it by breaking through the dykes that form the boundary. What reaches the surface will fuse, owing to the relief of pressure, and flow out so as to submerge the depressed portion. It is obvious from hydrostatics that it must rise to a level above the tops of the cracks, for a simple fracture would bring it nearly level with them, even if the margins were not bent down, and the curvature would be enough to send them far below the free surface. When the ejected matter solidified, which would not take long, a smooth surface would be formed. Here, again, we find a verification on the moon, for the large maria are extensive regions of great smoothness, and the regions between them are at higher levels. The bounding cracks would of course have been submerged below the outpour and become lost to sight for all time. What is particularly interesting about the maria, however, is that all the chief lie in a chain, forming the greater part of a circle, about 1000 miles in diameter, so that the suggested formation of a raised polygon is confirmed. The fact that they are darker than the average of the lunar surface, while the streaks are brighter, may perhaps be attributed to the different conditions of solidification inside a crack and in the open, or perhaps to the matter having come from a different depth.

The submersion of the matter around the edges of these polygons below hotter matter, far above its normal melting point, must have caused the depressed rocks there to melt again, or at least to soften. Now the density,

by hypothesis, increases with depth, and therefore the melted matter, being originally derived from a higher level in the crust, is lighter than its surroundings and tends to rise. The highest part of the polygon being the middle, on account of the curvature, the fused material would collect there. Thus the centre would become characterised by a greater depth of light matter than the edges; but if the average density down to a certain equipotential is excessively low at a place, a greater depth is required to give compensation. Hence, when the fused matter solidified again, even if the crust afterwards gave way under the tension involved, the centre would still remain elevated above the margins. Thus a permanent departure from sphericity would be produced. The region on the moon that has apparently been uplifted in this way is about 1000 miles across; a region on the earth of the same relative size would have a diameter of 3600 miles, about the width of Africa or North America. If such a process ever took place on the earth, we should therefore expect it to have led to the formation of elevated regions of similar size to our actual continents.

Before devoloping this hypothesis further let us consider certain further data about the moon which may be relevant. The lunar craters are the dominant physical feature of the continents, but they are almost absent from the maria. This is probably due to their having been formed before the maria; most of those that were originally present would then have been submerged in the outflowing lava and hidden, and only those of later origin would have examples in the maria. Streaks also are rare in the maria, for the same reason. It may be objected to the theory that after the outflow all tension in the crust would be relieved, and that therefore no cracks at all could be formed subsequently. The partial fusion of the surface by the hot liquid must, however, lead to the formation of a new hot solid surface, when tension could begin afresh, though with less violence than before. The origin of craters I have not attempted to account for. The rills may be analogous to rift valleys on the earth.

The origin of continents offers one of the most difficult problems in geophysics. Numerous attempts have been made to solve it, but none of the theories offered appear satisfactory. The tetrahedral theory is one of the best known; this starts with a newly solidified earth, and it is supposed that the contraction of the inner parts left the outer crust in a state in which it retained its original area, but had to collapse so as to accommodate itself to the reduced volume of the interior. The form adopted would, it was said, differ from the sphere in the direction of the regular solid with largest surface for the given volume, namely, the regular tetrahedron, and thus four continents would be formed, all at equal distances from one another. The

physical aspect of the theory has not been considered in detail. It derives some support from the fact that the actual distribution of land does bear some resemblance to a tetrahedron, though this is probably a peculiarity of the present time, and seems to have been widely departed from at some previous epochs. The fatal defect of the theory, it seems to me, is that a tetrahedral deformation does not correspond to a figure of equilibrium. It is known that for any such displacement the elevated parts must tend to come down again, since both gravity and the curvature of the elastic outer layer act so as to restore the original state. Instead of retaining the deformed figure, the earth would therefore oscillate about the spherical form till the oscillations were damped out, when symmetry would be restored. The shell, being too large for the interior, would then be unsupported and would collapse. A tetrahedral deformation cannot therefore be produced in this way; the way to render one possible is to have the continents free at their edges, so that curvature can take place in consequence of the natural cooling. The curved continent will then practically float on the heated interior, and any oscillation that may take place will merely move it up and down with the interior. Thus the process we have indicated is an essential preliminary of a tetrahedral deformation.

It has at various times been suggested that the birth of the moon may have had an important effect on the distribution of land and sea on the earth. This view must be examined with special care, because the hypothesis that the moon was formed from the earth by the disruptive action of the solar tides has acquired a moderate probability, though it cannot be regarded as demonstrated. It is known that the actual masses and motions of the earth and moon are such that if ever they formed one liquid body, this must have rotated in a period that would make the period of a free oscillation of tidal type very nearly coincident with the period of the solar semidiurnal tide at the same time. Accordingly the amplitude of this tide must have become very great, and it is certain that it must have become comparable with the diameter of the planet itself. If it ever exceeded a certain amount theoretically determinable, but not yet calculated, the planet must have broken up into two masses very like the earth and moon, and the rupture must have taken place in such a way that the subsequent development would be in the direction of the present state of the earth and moon; and there is little doubt now of the quantitative adequacy of tidal friction to produce the remainder of the changes, from then till now, in the motions of the two bodies. Such a series of agreements may be due to accidental coincidence, but the prior probability of this is small, and the theory may be accepted tentatively.

The importance of the effect of the rupture on geological history cannot

be determined till we know whether it took place before or after the commencement of solidification. The existence of a thin solid crust would not have affected the process much, as it would be very flexible; thus separation could have taken place at this stage. The crust would become broken into fragments; and the removal of a large quantity of surface matter from one side would leave a vast area with no light matter, which would, in the absence of redistribution, develop into the Pacific Ocean. This hypothesis is open to various objections, however. Though a thin solid crust would make little difference, the coincidence of periods would have been upset if the earth was not elsewhere almost wholly fluid. Now light solid fragments floating on a liquid interior, and largely confined to one side, would correspond to a first harmonic deformation, which has been shown to be unstable.* The fragments would therefore spread out again and redistribute themselves generally over the earth, so that the theory is not available as an explanation of the Pacific Ocean.

Again, the ellipticity of the earth at this time must have been at least thirty times what it is now, and the formation of a new crust would only have taken a geologically short time. In the subsequent evolution, the equator must therefore have contracted by an amount of at least 2000 km., which would have caused the formation of mountain chains across the equator, running north and south, and far exceeding in size any known mountains. The solidification of the earth must then have occurred when the moon had attained a considerable distance, and the earth's speed of rotation had accordingly become much diminished. This is confirmed by the fact that the moon itself seems to have been largely fluid until it reached a distance of about 150,000 km. from the earth, so that this criticism is answerable. A further criticism is that even if the Pacific Ocean were accounted for in this way, a similar explanation would not be available for the asymmetry that exists on Mars; for Mars has certainly not acquired a satellite by fissure, and, even if it had, the equator would run through the middle of the ocean produced, whereas the dark regions on Mars, which are believed to be the seas, are nearly confined to the southern hemisphere. There must therefore be some alternative reason for an asymmetrical distribution of oceans.

Summary.

On the hypothesis that the earth was formerly molten, a theory of its thermal history is developed, based largely on the numerical results of

* J. H. Jeans, "Gravitational Instability and the Figure of the Earth," 'Roy. Soc. Proc.,' A, vol. 93, pp. 413–417 (1917).

Arthur Holmes. In the main, Holmes's inferences are confirmed. It is found that the available compression is probably sufficient to account for all existing mountains, and an explanation of the Pacific type of mountain range is based on the greater cooling and consequent strength of the sub-oceanic rocks. A physical interpretation of the results of investigations of isostasy is offered, and is used in a discussion of the compensation of oceanic inequalities. It is shown that the inequality that gives rise to the land and water hemispheres is necessarily compensated, and that the oceans are also in the main probably compensated. An explanation of the fact that the great oceans have regions of smaller depth in the middle is founded on the greater cooling of sub-oceanic rocks. This theory requires that the deeper parts of the oceans should be associated with low values of gravity, which is capable of direct verification.

The stresses that would be set up in the earth's crust soon after solidification are discussed, and the results appear to agree with observations of the present state of the moon. A theory of the origin of continents is developed from this, and appears to have certain important advantages over those at present current.

On the Absorption of Light by Electrically Luminescent Mercury Vapour.

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[PLATE 3.]

Introductory.

Some of the earliest experiments on the absorption of light by electrically luminous gases were carried out by Pflüger,* who, in 1907, investigated the absorption and reversal of the hydrogen lines by luminous hydrogen. He used a condensed discharge in a three-electrode tube, in which a short constriction provided the source of radiation and the wider and longer part the absorbing column. He succeeded in reversing H_α. This work was followed up by Landenburg and Loria,† who reversed H_α and H_β. In the same year

* 'Ann. der Phys.,' vol. 24, p. 515 (1907).
† See Wood's 'Physical Optics,' p. 434.